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On the budget terms of the double averaged turbulent stress transport equations in porous media

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Abstract

To provide information for modeling turbulent flows in porous media, LES (large eddy simulation) of fully developed flows in porous media is performed. The numerical scheme used for the LES is the D3Q27 (three dimensional 27 discrete velocity) MRT (multiple relaxation time) LBM (lattice Boltzmann method) coupled with the WALE (wall-adapting local eddy-viscosity) sub-grid-scale turbulence model. The considered porous medium configurations are square rod arrays, cube arrays and BCC (body centered cube) foams. Using the statistical data by the LES, the behaviors of the budget terms of the double averaged turbulent stress transport equations are investigated carefully for modeling those transport equations.

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Keywords: Large eddy simulation; Lattice Boltzmann method;Turbulence modeling; Porous medium; Double averaging

1. Introduction

Many industrial and environmental applications include flows inside porous materials and thus analyzing porous medium flows has progressively become important. In applying CFD (computational fluid dynamics) to porous medium flows, there can be two types of computations: microscopic and macroscopic computations. The former faithfully treats the flow geometry in the porous medium. However, it is readily understood that even with the modern computer resources, a microscopic computation of a porous medium flow is prohibitively expensive

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particularly for engineering CFD. Therefore, the VAT (volume averaging theory) [1] is usually applied to treat momentum equations of flows in porous media. That is the basis of the macroscopic computation. Since flows become turbulent even in porous media, turbulence models are required for those flows when a statistical treatment by the VAT is applied. Due to the volume averaging of the Reynolds averaged momentum equation, one can derive two unknown covariances: the dispersive covariance and the volume averaged Reynolds stress. The volume averaged Reynolds stress can be further divided into two parts: the macro-scale and micro-scale Reynolds stresses. To solve the volume averaged turbulence equations in porous media thus needs modeling those covariances. However, there is not enough information for those covariant processes. Accordingly, people applied phenomenological ideas to the modeling and thus almost all studies on modelling turbulent flows in porous media significantly drop and ignore many unknown correlations emerging in the stress transport equations. Consequently, those models were based on the relatively crude approximations in applying the eddy viscosity turbulence model [2,3]. However, when one consider to treat flows inside and/or around porous media, more precise modelling for turbulence is required and the present authors tried to develop a turbulence model treating each covariance independently by applying the second moment closure to the macro-scale Reynolds stress [4]. Without any information for the behaviors of the budget terms in the transport equation of each covariance, it is indeed difficult to evaluate each model term. Therefore, in this study, to develop the model more logically, it is attempted to discuss the behaviors of the budget terms by performing microscopic computations for turbulent porous medium flows. The scheme used for the microscopic computations is the LES (large eddy simulation) with the WALE (wall-adaptive local eddy-viscosity) model [5] by the D3Q27 (3 dimensional 27 discrete velocity) MRT (multiple relaxation time) LBM (lattice Boltzmann method) [6].

2. LES by the MRT LBM

The lattice Boltzmann equation can be obtained by discretizing the velocity space of the Boltzmann equation into a finite number of discrete velocities $\boldsymbol{\xi}_\alpha \; \{\alpha = 0, \cdots, Q-1\}$. The MRT LBM [6] transforms the distribution function $\mathbf{f}$ in the velocity space to the moment space by the transformation matrix $\mathbf{M}$. Since the moments of the distribution function correspond directly to flow quantities, the moment representation allows us to perform the relaxation processes with different relaxation-times according to different time-scales of various physical processes. The evolution equation is thus written as

$$\left|\mathbf{f}(\mathbf{x}+\xi_\alpha \delta t, t+\delta t)\right\rangle - \left|\mathbf{f}(\mathbf{x},t)\right\rangle = -\mathbf{M}^{-1}\hat{\mathbf{S}}\left[\left|\mathbf{m}(\mathbf{x},t)\right\rangle - \left|\mathbf{m}^{eq}(\mathbf{x},t)\right\rangle\right] + \mathbf{M}^{-1}\left(\mathbf{I}-\frac{\hat{\mathbf{S}}}{2}\right)\mathbf{M}\left|\mathbf{F}\right\rangle \delta t \,, \tag{1}$$

where the notations such as $|\mathbf{f}\rangle$ is $|\mathbf{f}\rangle = (f_0, f_1, \cdots, f_{Q-1})^T$, $\mathbf{I}$ is the identity matrix and δt is the time step. The term $\mathbf{F}$ is an external body force. The matrix $\mathbf{M}$ is a $Q \times Q$ matrix which linearly transforms the distribution function to the moment: $|\mathbf{m}\rangle = \mathbf{M} \cdot |\mathbf{f}\rangle$ and its collision matrix $\hat{\mathbf{S}}$ is diagonal:

$$\hat{\mathbf{S}} \equiv diag(0,0,0,0,s_4,s_5,s_5,s_7,s_7,s_7,s_{10},s_{10},s_{10},s_{13},s_{13},s_{13},s_{16},s_{17},s_{18},s_{18},s_{20},s_{20},s_{20},s_{23},s_{23},s_{23},s_{26}) \,, \tag{2}$$

for the D3Q27 discrete velocity model. The relaxation parameters used are

$$s_4 = 1.54,\; s_7 = s_5,\; s_{10} = 1.5,\; s_{13} = 1.83,\; s_{16} = 1.4,\; s_{17} = 1.61,\; s_{18} = s_{20} = 1.98,\; s_{23} = s_{26} = 1.74 \,, \tag{3}$$

which were optimized by [8]. The relaxation parameter s_5 is related to the fluid viscosity, and thus for the LES, it is $\nu + \nu_{SGS} = c_s^2(1/s_5 - 1/2)\delta t$ where $c_s = 1/\sqrt{3}$ is the sound speed in the D3Q27 LBM.

The sub-grid-scale model applied is the WALE model [5] and its eddy viscosity is described as

$$\nu_{SGS} = (C_w\Delta)^2 \frac{(S_{ij}^d S_{ij}^d)^{3/2}}{\left(S_{ij}S_{ij}\right)^{5/2} + (S_{ij}^d S_{ij}^d)^{5/4}} \,, \tag{4}$$

where $S_{ij} = (\partial\hat{u}_i/\partial x_j + \partial\hat{u}_j/\partial x_i)/2$, $S_{ij}^d = \{(\partial\hat{u}_i/\partial x_j)^2 + (\partial\hat{u}_j/\partial x_i)^2\}/2 - (\partial\hat{u}_k/\partial x_k)^2\delta_{ij}/2$ and the value $\hat{u}_i$ is the filtered velocity. In the present study, the eddy viscosity coefficient applied is $C_w = 0.1$.

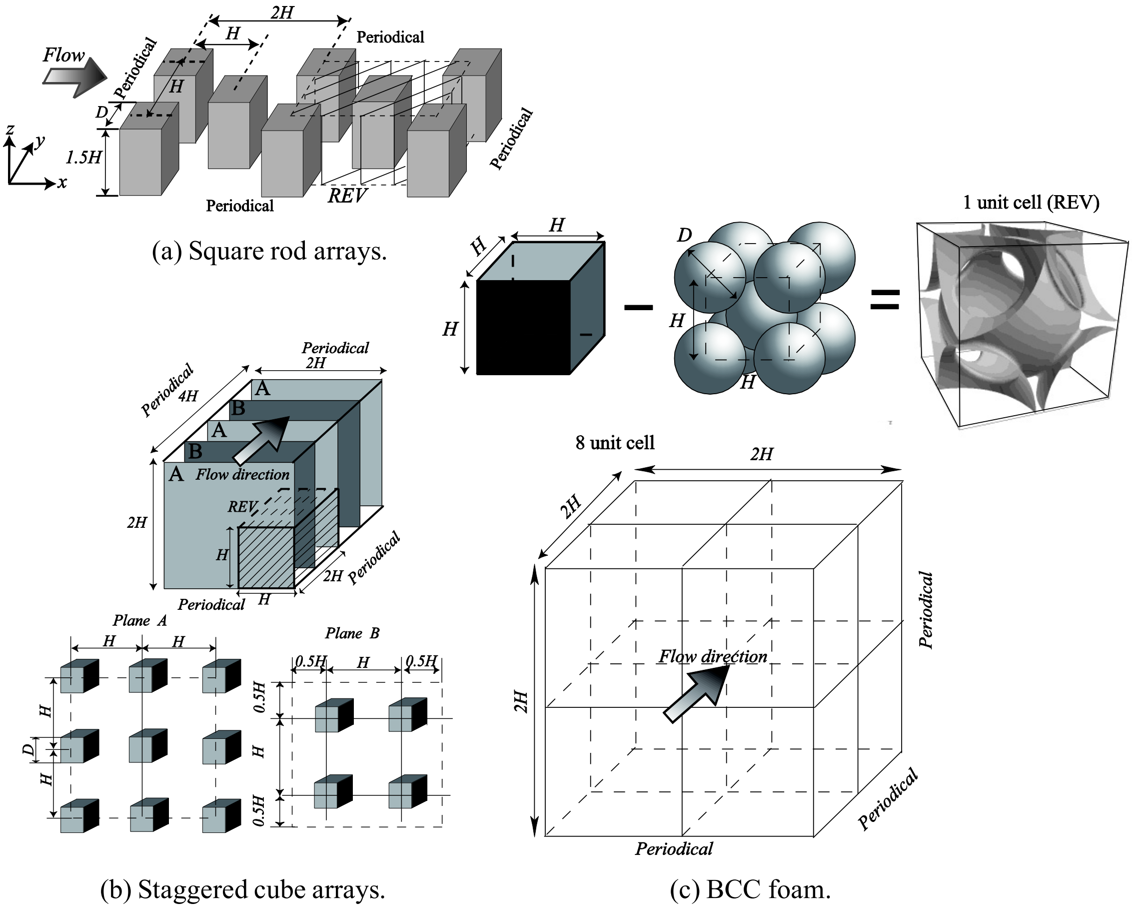

Fig. 1. Model geometries and computational domains of porous media.

The computational domain considered for the porous media are $4H(x)\times H(y)\times 1.5H(z)$, $4H\times 2H\times 2H$ and $2H\times 2H\times 2H$, respectively in square rod arrays, staggered cube arrays and BCC (body centered cube) foam as shown in Fig. 1. After performing grid sensitivity tests, uniform computational grids of $481\times 121\times 180$, $321\times 161\times 161$ and $256\times 256\times 256$ are used respectively for those geometries. Periodic boundary conditions are applied for each domain surfaces though a pressure difference is imposed in the streamwise direction. To describe the wall surfaces, the interpolated bounce-back boundary method is applied for the density distribution functions. The computed flows are in the range of 1000-3500 of the pore Reynolds number.

3. Double averaged equations

The volume averaged values are called the *superficial* and *intrinsic* (fluid phase) averaged values and defined as

$$\langle \phi \rangle = \frac{1}{\Delta V}\int_{\Delta V_f} \phi dv , \quad \langle \phi \rangle^f = \frac{1}{\Delta V_f}\int_{\Delta V_f} \phi dv , \tag{5}$$

where ΔV and ΔV_f are the REV (representative elementary volume) and the volume of the fluid phase contained within ΔV, respectively. Between them, the relation: $\langle \phi \rangle = \varphi \langle \phi \rangle^f$ exists with the porosity of the porous medium: $\varphi = \Delta V_f / \Delta V$. The volume averaging of derivatives is expressed as

$$\left\langle \frac{\partial \phi}{\partial x_k} \right\rangle = \frac{\partial \langle \phi \rangle}{\partial x_k} + \frac{1}{\Delta V} \int_A n_k ds \,, \tag{6}$$

where A, n_k are the superficial area of the solid phase and its unit normal vector pointing outward from the fluid to the solid phase, respectively. The dispersion is $\tilde{\phi} = \phi - \langle \phi \rangle^f$ and the interchangeable rules [7] between the Reynolds and volume averaged values are $\overline{\langle \phi \rangle^f} = \langle \overline{\phi} \rangle^f$, $\langle \phi' \rangle^f = \langle \phi \rangle^{f\prime}$, $\tilde{\phi}' = \tilde{\phi}'$, $\overline{\tilde{\phi}} = \tilde{\overline{\phi}}$. Then the double (volume and Reynolds) averaged momentum equation for homogeneous porous media is

$$\frac{\partial \langle \overline{u}_i \rangle^f}{\partial t} + \langle \overline{u}_j \rangle^f \frac{\partial \langle \overline{u}_i \rangle^f}{\partial x_j} = -\frac{1}{\rho} \frac{\partial \langle \overline{p} \rangle^f}{\partial x_i} + \nu \frac{\partial^2 \langle \overline{u}_i \rangle^f}{\partial x_j^2} - \overline{f}_i - \frac{\partial \left(\langle \tilde{\overline{u}}_i \tilde{\overline{u}}_j \rangle^f + \langle \overline{u_i' u_j'} \rangle^f \right)}{\partial x_j} \,, \tag{7}$$

where f_i is the drag force term. The terms $\mathscr{T}_{ij} = \langle \tilde{\overline{u}}_i \tilde{\overline{u}}_j \rangle^f$ and $R_{ij}^A = \langle \overline{u_i' u_j'} \rangle^f$ are the dispersive covariance and the volume averaged (total) Reynolds stress. The latter can be further decomposed as

$$R_{ij}^A = \underbrace{\overline{\langle u_i' \rangle^f \langle u_j' \rangle^f}}_{R_{ij}^M} + \underbrace{\langle \overline{\tilde{u}_i' \tilde{u}_j'} \rangle^f}_{R_{ij}^m} \,, \tag{8}$$

where R_{ij}^M and R_{ij}^m are namely the macro-scale and micro-scale Reynolds stresses, respectively. The transport equations of those stresses in fully developed homogeneous porous media are

$$\frac{D\mathscr{T}_{ij}}{Dt} = \Psi_{ij} - P_{ij}^d + P_{ij}^t + \overline{f}_i \langle \hat{\overline{u}}_j \rangle^f + \overline{f}_j \langle \hat{\overline{u}}_i \rangle^f + \mathscr{E}_{ij} \,, \tag{9}$$

$$\frac{DR_{ij}^M}{Dt} = \phi_{ij}^M - P_{ij}^t + C_{ij}^t - F_{ij}^t - \varepsilon_{ij}^M = 0 \,, \quad \frac{DR_{ij}^m}{Dt} = \phi_{ij}^m + P_{ij}^d - C_{ij}^t + F_{ij}^t - \varepsilon_{ij}^m = 0 \,, \tag{10}$$

where

$$P_{ij}^d = -\left\langle \overline{\tilde{u}_i' \tilde{u}_k'} \frac{\partial \tilde{\overline{u}}_j}{\partial x_k} \right\rangle^f - \left\langle \overline{\tilde{u}_j' \tilde{u}_k'} \frac{\partial \tilde{\overline{u}}_i}{\partial x_k} \right\rangle^f - \left\langle \overline{\tilde{u}_i' \langle u_k' \rangle^f} \frac{\partial \tilde{\overline{u}}_j}{\partial x_k} \right\rangle^f - \left\langle \overline{\tilde{u}_j' \langle u_k' \rangle^f} \frac{\partial \tilde{\overline{u}}_i}{\partial x_k} \right\rangle^f , \quad P_{ij}^t = -\left\langle \overline{\tilde{u}_k' \tilde{\overline{u}}_j \frac{\partial \langle u_i' \rangle^f}{\partial x_k}} \right\rangle^f - \left\langle \overline{\tilde{u}_k' \tilde{\overline{u}}_i \frac{\partial \langle u_j' \rangle^f}{\partial x_k}} \right\rangle^f , \tag{11}$$

$$\mathscr{E}_{ij} = 2\nu \left\langle \frac{\partial \tilde{\overline{u}}_i}{\partial x_k} \frac{\partial \tilde{\overline{u}}_j}{\partial x_k} \right\rangle^f , \quad \varepsilon_{ij}^M = 2\nu \overline{\frac{\partial \langle u_i' \rangle^f}{\partial x_j} \frac{\partial \langle u_j' \rangle^f}{\partial x_i}} , \quad \varepsilon_{ij}^m = 2\nu \left\langle \overline{\frac{\partial \tilde{u}_i'}{\partial x_j} \frac{\partial \tilde{u}_j'}{\partial x_i}} \right\rangle^f , \tag{12}$$

and $\langle \hat{\overline{u}}_i \rangle^f$, Ψ_{ij}, ϕ_{ij}^M, ϕ_{ij}^m, C_{ij}^t and F_{ij}^t are the relative velocity to the solid phase, pressure-dispersive-velocity-strain, macro-scale pressure-strain, micro-scale pressure-strain, macro-micro turbulence cascade and turbulent drag force terms, respectively. In the fully developed homogeneous porous media presently discussed, R_{ij}^M vanishes and thus ϕ_{ij}^M, C_{ij}^t and F_{ij}^t also vanish except for the highly porous cases of the square rod array flows as discussed in the next section 4.

4. Results and discussions

By analyzing the simulation data, the behavior of the budget terms shown in Eqs.(9)-(12) against the porosity is investigated. Since the macro-scale Reynolds stress is virtually zero in the fully developed porous medium flows tested in this study, the budget terms of the micro-scale Reynolds stress are focused on. (Note that in the square rod array flows, it can be detected that a very small level of the cross-beamwise macro-scale Reynolds stress component is produced by the vortex shedding.) Also, the turbulent shear production P_{ij}^t becomes negligibly small in all the cases, and hence Fig. 2 only shows the behaviors of the dispersive shear production P_{ij}^{d} . In the square rod arrays case, Fig. 2(a), $P_{33}^{d} = 0$ because of zero spanwise gradient of the dispersive velocity. At $\varphi = 0.5$, P_{11}^{d} and P_{22}^{d} are nearly the same and as the increase of the porosity, they decrease while at $\varphi > 0.75$ the decreasing rate of P_{22}^{d} becomes lower. In the cube array flows of Fig. 2(b), P_{11}^{d} is always dominant while the other components are marginally small. In the BCC foam flows, P_{ij}^{d} is nearly isotropic as in Fig. 2(c). It is thus understood that the behavior of P_{ij}^{d} , which is an important term to be modeled, is so diverse depending on the structure of the porous media.

In the fully developed porous media, the following relations come out from the transport equations of the dispersive and micro-scale turbulent energy which are derived by Eqs.(9) and (10):

$$\frac{D\mathscr{T}_{kk}/2}{Dt} \approx -P_{kk}^{d}/2 + \overline{f}_k \left\langle \hat{\bar{u}}_k \right\rangle^f - \mathscr{E}_{kk}/2 = 0\,, \quad \frac{DR_{kk}^{m}/2}{Dt} \approx P_{kk}^{d}/2 - \varepsilon_{kk}^{m}/2 = 0\,, \tag{13}$$

since the macro-scale turbulence and P_{kk}^{t} are negligible. The drag force term in Eq.(13) balances with the sum of the dispersive shear production and the dissipation terms. The behaviours of those drag force and the dispersive shear production terms are indicated in Fig. 3. Interestingly, the profiles of the dispersive production and the drag force terms are fairly synchronized in each case. This fact indicates that the majority of the dispersive kinetic energy, which is proportional to the drag force, transfers to the micro-scale turbulence energy via the dispersive shear production term P_{kk}^{d} . Consequently, with the relation: $P_{kk}^{d} \approx \varepsilon_{kk}^{m}$, the transferred energy dissipates in the micro-scale turbulence. For the modeling issue, it is suggested that the dispersive shear production term may be scaled by the drag force term $\overline{f}_k \left\langle \hat{\bar{u}}_k \right\rangle^f$ which is usually modeled using the Darcy-Forchheimer model [1].

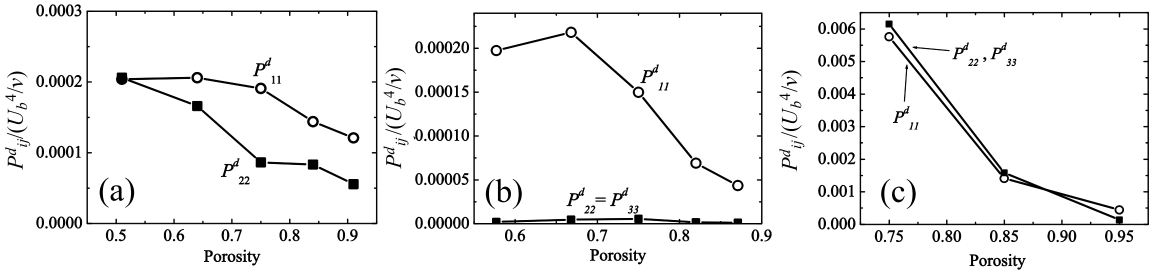

Fig. 2. Dispersive shear production terms in the porous media: (a) square rod arrays; (b) staggered cube arrays; (c) BCC foam.

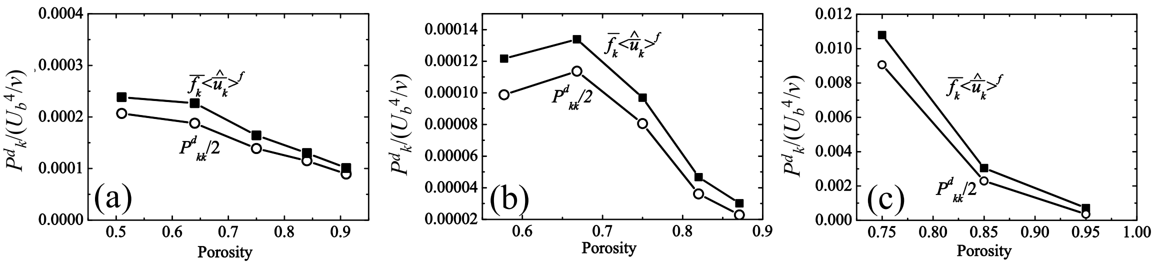

Fig. 3. Production and drag terms in the porous media: (a) square rod arrays; (b) staggered cube arrays; (c) BCC foam.

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